VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

JEE Advanced Home Practice Test -8 | Paper -1 | JEE 2024

Date: 13/05/2024   **Maximum Marks: 186**

Duration : 3.0 Hours

General Instructions

1. The question paper consists of 3 Subjects (Subject I: **Physics**, Subject II: **Chemistry**, Subject III: **Mathematics**). Each Part has **Three** sections (Section 1, Section 2 and Section 3).

2. **Section 1** contains **4 Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

3. **Section 2** contains **8 Multiple Correct Answer Type Questions.** Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

4. **Section 3** contains **6 Numerical Value Type Questions**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

5. For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code, Roll No.** and **Group** properly in the space given in the ANSWER SHEET.

Name of the Candidate (In CAPITALS) : _

Roll Number : _ .

OMR Bar Code Number : _

Candidate's Signature : _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _   Invigilator's Signature _ _ _ _ _ _ _ _ _ _ _ _ _

MARKING SCHEME

SECTION-1 | (Maximum Marks: 12)

➢ This section contains **Four (04) Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

➢ Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

➢ For each question, choose the option corresponding to the correct answer.

➢ Answer to each question will be evaluated according to the following marking scheme.

Full Marks	:	+3 If ONLY the correct option is chosen.
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	−1 In all other cases.

SECTION-2 | (Maximum Marks: 32)

➢ This section consists of **Eight (08)** Questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

➢ Answer to each question will be evaluated according to the following marking scheme:

|		
Full Marks	:	+4 If only (all) the correct option(s) is(are) chosen.
Partial Marks	:	+3 If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	+2 If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks chosen,	:	+1 If two or more options are correct but ONLY one option is and it is a correct option.
Zero Mark	:	0 if none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	−1 In all other cases.

SECTION-3 | (Maximum Marks: 18)

➢ This section contains **Six (06) Numerical Value Type Questions**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

➢ Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3 **ONLY** if the correct numerical value is entered.
Zero Mark	:	0 In all other cases.

SUBJECT I : PHYSICS   **62 MARKS**

Section-1 | Single Correct Type

This Section contains **4 Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

**1.** Consider a light beam incident from air to a glass slab at Brewster's angle as shown in figure.
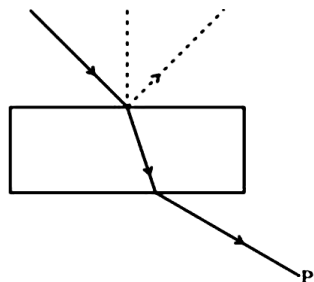
A polaroid is placed in the path of the emergent ray at point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid.

(A) For a particular orientation there shall be darkness as observed through the polaroid

(B) The intensity of light as seen through the polaroid shall be independent of the rotation

(C) The intensity of light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid

(D) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid

2. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, and if $\sigma = \dfrac{1}{2\pi}\mu c/m^2$, $r_0 = 1m$.

(A) $Q = 3\mu C$ **(B)** $\lambda = 1\mu C/m$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$ **(D)** $E_2(r_0/2) = 4E_3(r_0/2)$

3. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as $T = T_0(1+\beta t^2)$. Where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of metal is:

(A) $\dfrac{P/2}{\sqrt{\beta T_0 (T - T_0)}}$

(B) $\dfrac{P}{4\beta T_0 \sqrt{(T - T_0)}}$

(C) $\dfrac{P(T - T_0)^{1/2}}{\beta T_0}$

(D) $\dfrac{P(T - T_0)^{3/2}}{\beta T_0}$

4. In a radioactive sample $^{40}_{19}K$ nuclei either decays into stable $^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable $^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei are produced by the $^{40}_{19}K$ nuclei only. In time $t \times 10^9$ years, if the ratio of the sum of stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei to the radioactive $^{40}_{19}K$ nuclei is 9, the value of t will be: [Given : $\ln 10 = 2.3$]

(A) 9.2   (B) 4.6   (C) 1.15   (D) 2.3

SPACE FOR ROUGH WORK

Section-2 | Multiple Correct Type

This Section contains **8 Multiple Correct Answer Type Questions.** Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

5. A glass capillary tube closed at upper end has internal and external radius as r and R respectively. The tube is held vertical with its lower end touching the surface of water. Assume that water perfectly wets the glass. Then, which of the following statement(s) is(are) correct?

(Atmospheric pressure is P_0, surface tension of water is T, density of water is ρ, mass of empty tube is M).

(A) For water in it to rise upto height $h(< L)$, length (L) of such tube is $\left(h + \dfrac{2P_0 rh}{2T - \rho grh} \right)$

(B) For water in it to rise upto height $h(< L)$, length (L) of such tube is $\left(h + \dfrac{P_0 rh}{2T - \rho grh} \right)$

(C) The vertical force needed to hold tube in this position will be

$$F = Mg + \pi P_0 \left[R^2 - \frac{Lr^2}{L - h} \right] + 2\pi (R + r)T$$

**(D)** The vertical force needed to hold tube in this position will be

$$F = Mg + \pi P_0 \left[R^2 - \frac{Lr^2}{L + h} \right] + 2\pi (R + r)T$$

6. A circular conducting loop of radius R and resistance per unit length λ is pulled out from the region of uniform magnetic field with constant velocity v. The situation shown in the figure corresponds to time $t = 0$. Which of the following statement(s) is(are) correct?
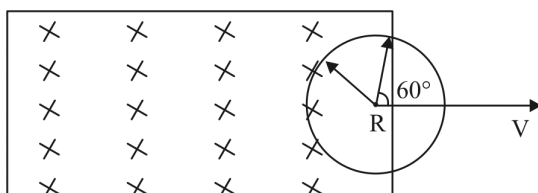
(A) Just after $t = 0$ i.e., the motion starts, the induced current in the loop is $\dfrac{\sqrt{3} Bv}{2\pi\lambda}$

(B) Current will be induced in the loop for $\dfrac{3R}{2v}$ sec

(C) At any time t, the current induced in the loop is given by $\dfrac{Bv\sqrt{3R^2 - 4v^2 t^2 + 4Rvt}}{\lambda \times 2\pi R}$ and is in clockwise direction

(D) Induced current is in clockwise direction for $t = 0$ to $\dfrac{R}{2v}$ and thereafter it becomes in anticlockwise direction

7. Two particles A and B of mass m and $2m$ respectively, initially at a distance a from each other, are released from rest. The particles move towards each other due to their mutual gravitational attraction. Neglect the forces on the particles due to all other bodies. At the instant, the distance between the particles is $\dfrac{a}{2}$, which of these option(s) is/are correct? [G denotes the gravitational constant]

    (A) The velocity of A is $\sqrt{\dfrac{8Gm}{3a}}$

    (B) The acceleration of A is $\dfrac{4Gm}{a^2}$

    (C) The combined kinetic energy of the two particle is $\dfrac{4Gm^2}{a}$

    (D) The distance that A has moved until this instant is $\dfrac{a}{3}$

8. A cube of side length $2a$, has volumetric charge density ρ. A Gaussian spherical surface of radius $= r$, is taken for different situation. Which of the following option(s) is(are) correct?

    (A) If center of cube and centre of spherical surface coincide, then for $r = 1.25\,a$, electric flux through spherical surface is $\dfrac{\rho}{\in_0}\dfrac{13}{6}\pi a^3$

    (B) If center of cube and center of spherical surface coincide, then for $r = 0.5\,a$, electric flux through spherical surface is $\dfrac{\rho\pi a^3}{6\in_0}$

    (C) If center of cube and center of spherical surface coincide, then for $r = \sqrt{2}\,a$, electric flux through spherical surface is $\dfrac{8\rho a^3}{\in_0}$

    (D) If center of spherical surface is at one of corner of the cube, then for $r = a$, electric flux through spherical surface is $\dfrac{\rho\pi a^3}{6\in_0}$

9. n moles of an ideal gas undergoes an isothermal process at temperature T. $P - V$ graph of the process is as shown in the figure. A point $A(V_1, P_1)$ is located on the $P - V$ curve. Tangent at point A, cuts the V-axis at point B. AO is the line joining the point A to the origin O of PV diagram. AO is also the perpendicular to the tangent at point A. BC is perpendicular to OB. Then which of the following statement(s) is(are) correct?

    (A) The coordinates of point C are $\left(2V_1, \dfrac{P_1}{2}\right)$

    (B) The coordinates of point C are $\left(\dfrac{3}{2}V_1, \dfrac{2P_1}{3}\right)$

    (C) Area of the triangle AOB is equal to nRT

   **(D)** Area of the triangle AOB is equal to $\dfrac{nRT}{2}$
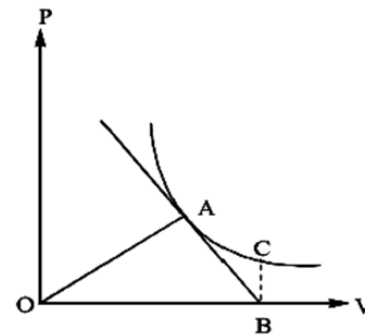
10. A spherical surface separates air and medium for which $\mu = 1.61$ for violet and $\mu = 1.60$ for red color. A paraxial beam parallel to optic axis is incident on the surface as shown. The distance between point of convergence for violet and red color is Δf. Find the magnitude of Δf (in *cm*).

(A) $\Delta f = 0.27 \, cm$

(B) Point of convergence for red is closer to optical centre than that for violet

(C) Point of convergence for violet is closer to optical centre than that for red

(D) $\Delta f = 0.84 \, cm$
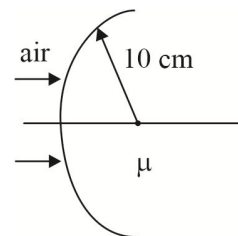
11. Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity X as follows: $[\text{position}] = [X^{\alpha}]$; $[\text{speed}] = [X^{\beta}]$; $[\text{acceleration}] = [X^{p}]$; $[\text{Linear momentum}] = [X^{q}]$; $[\text{force}] = [X^{r}]$. Then:

(A) $\alpha + \beta = 2q$ **(B)** $p + q - r = \beta$ **(C)** $p - q + r = \alpha$ **(D)** $p + q + r = \beta$

12. Two identical moving coil galvanometers have $10 \, \Omega$ resistance and full-scale deflection at $2\mu A$ current. One of them is converted into a voltmeter of $200 \, mV$ full scale reading and the other into an Ammeter of $10 \, mA$ full scale current using appropriate resistor. These are then used to measure the voltage and current in the Ohm's law experiment with $R = 1000 \, \Omega$ resistor by using an ideal cell. Which of the following statement(s) is/are correct?

(A) Resistance of voltmeter is $100 \, k \, \Omega$

(B) Resistance of ammeter is approximately 2 milli-ohm

(C) If non-ideal cell of internal resistance $10 \, \Omega$ is used, the measured valued will decrease

(D) Measured value of resistance is between $989 \, \Omega$ to $991 \, \Omega$

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SPACE FOR ROUGH WORK

Section-3 | Numerical Value Type

This **Section** contains **6 Numerical Value Type Questions**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

13. A small ball of mass 2 *kg* is slowly pulled up to the top of fixed rough hemisphere as shown in figure. If $\mu = 0.25$ and $R = 20\,cm$, the work done by applied force is: $[g = 10\,m/s^2]$
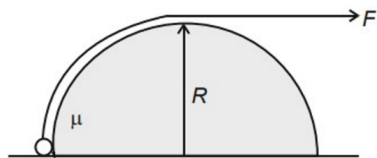
14. Two rigid rods of same material and of length L and $\dfrac{L}{2}$ are placed in equilibrium on a smooth horizontal plane at temperature T as shown in figure. Springs are in their natural length. If the temperature is increased by ΔT, total amount of energy stored in springs is $n\,k\alpha^2 L^2 \Delta T^2$, where k is the spring constant of spring **(A)** and α is the coefficient of linear expansion of material. Find the value of *n*. [Neglect the change in spring constant due to increase in temperature]
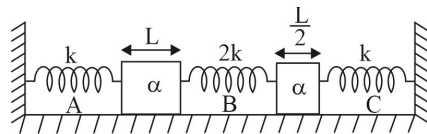
15. Two cars *A* and *B* are approaching a cross road junction *P* as shown. At an instant when *A* is 180 *m* and *B* is 255 *m* away from *P*, A momentarily blows a horn of frequency 2.94 *Hz*. Find corresponding frequency observed by *B*? $[V_{sound} = 300\,m/s]$
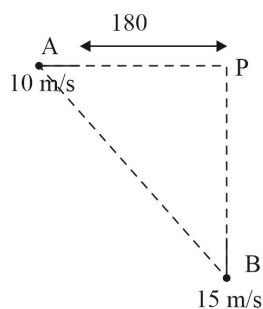
16. A parallel plate condenser, with plate area A and distance between plates d, is filled with a medium whose permittivity varies as;

$$\in(x) = \in_0 + \beta \in_0 x/d \qquad 0 < x < \dfrac{d}{2}$$

$$\in(x) = \in_0 + \beta \in_0 \dfrac{(d-x)}{d} \qquad \dfrac{d}{2} < x < d$$

x is the distance from one of the plates. For what value of β would the capacity of the condenser be $\dfrac{1}{2\ln 2}$ times of that when it is completely filled with a uniform dielectric having dielectric constant β ?

17. A piece of ice (specific heat capacity $= 2100\,J\,kg^{-1}\,{}^{\circ}C^{-1}$ and latent heat $= 3.36\times10^{5}\,J\,kg^{-1}$) of mass m gram is at $-5{}^{\circ}C$ at atmospheric pressure. It is given $420\,J$ of heat so that the ice starts melting. Finally, when the ice-water mixture is in equilibrium, it is found that $1g$ of ice has melted. Assuming there is no other heat exchange in the process, the value of m is _____.

18. Consider the shown arrangement of a plane mirror and a concave mirror ($f_m = 20\,cm$) placed at distance of $50\,cm$ of shown. If point object S is placed in between, then numerical value of Y-coordinate of image formed by plane mirror (consider 1st reflection from concave mirror and then by plane mirror) is found to be $5N\,cm$. Find N _____.
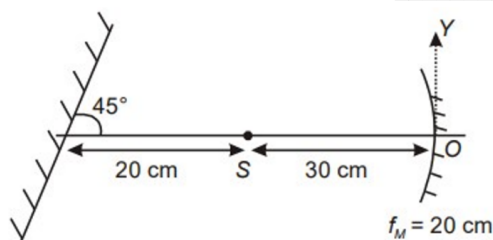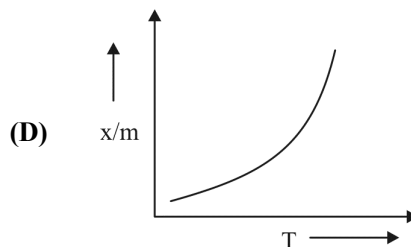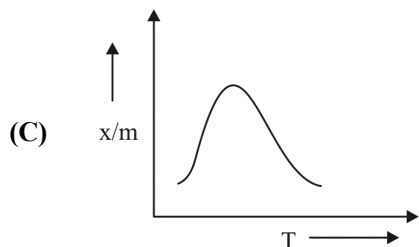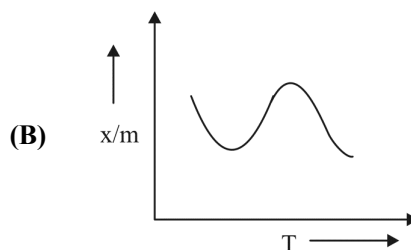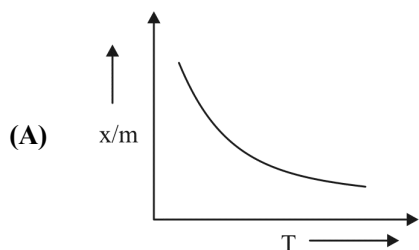

SPACE FOR ROUGH WORK

| **SUBJECT II : CHEMISTRY** | **62 MARKS** |

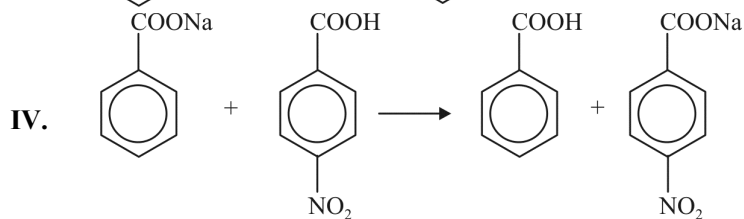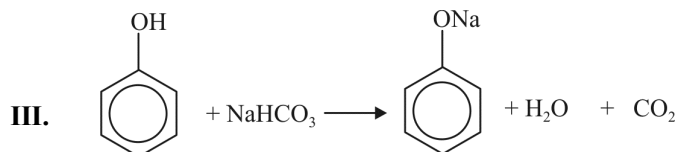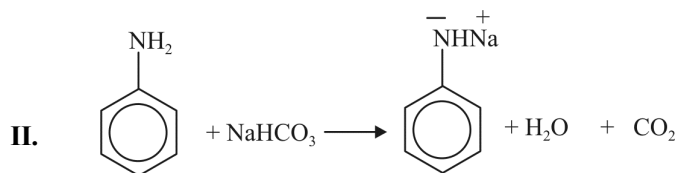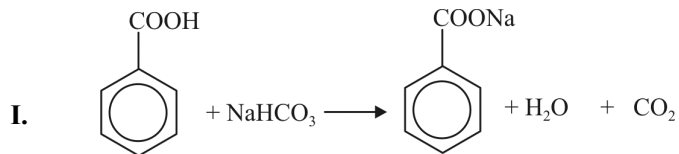## Section-1 | Single Correct Type

This Section contains **4 Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

1. The compound present in borax bead is:

(A) B_2O_3 (B) $NaBO_2$ (C) $NaBO_3$ (D) $NaBO_2 + B_2O_3$

2. Bauxite, siderite and argentite are respectively:
- (A) Sulphide, Oxide and carbonate ore
- (B) Oxide, carbonate and sulphide ore
- (C) Oxide, oxide and sulphide ore
- (D) Oxide, sulphide and oxide ore

3. Which of the following graph represents the variation of amount of chemisorption of a gas by a solid with temperaure under constant pressure?

SPACE FOR ROUGH WORK

4. Which of the following reactions are feasible (practically possible)?

I.   COOH + NaHCO$_3$ ⟶ COONa + H$_2$O + CO$_2$

II.   NH$_2$ + NaHCO$_3$ ⟶ $\overset{-}{\text{N}}\text{H}\overset{+}{\text{Na}}$ + H$_2$O + CO$_2$

III. OH + NaHCO$_3$ ⟶ ONa + H$_2$O + CO$_2$

IV.   COONa + COOH(NO$_2$) ⟶ COOH + COONa(NO$_2$)

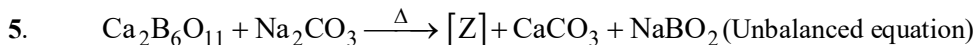| (A)	I, II	(B)	II, III	(C)	III, IV	(D)	I, IV

SPACE FOR ROUGH WORK

Section-2 | Multiple Correct Type

This Section contains **8 Multiple Correct Answer Type Questions.** Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

5. $Ca_2B_6O_{11} + Na_2CO_3 \xrightarrow{\Delta} [Z] + CaCO_3 + NaBO_2$ (Unbalanced equation)

    Correct statement for [Z].

    (A) Structure of anion of crystalline (Z) has two boron atom sp^3 hybridized and other two boron atoms sp^2 hybridized.

    (B) (Z) with NaOH(aq.) gives a compound which on reaction with H_2O_2 in alkaline medium yields a compound used as brightener in soaps.

    (C) Hydrolysis of (Z) with HCl or H_2SO_4 yields a compound which on reaction with HF gives fluoroboric acid.

    (D) (Z) on heating with cobalt salt in oxidizing flame gives pink coloured bead.

6. Which of the following statements regarding d-block elements are true?

    (A) the color of anhydrous $CuSO_4$ is blue

    (B) "splitting of silver" can be prevented by converging the surface of molten silver with charcoal

    (C) Cr(VI) in the form of dichromate in acidic medium is a strong oxidizing agent whereas MoO_3 and WO_3 are not.

    (D) Lanthanum is first element of third transition series.

7. The value of $\Delta H_{transition}$ of $C(graphite) \rightarrow C(diamond)$ is 1.9 kJ/mol at 25°C and entropy of graphite is higher than entropy of diamond. This implies that:

    (A) C(diamond) is more thermodynamically stable than C(graphite) at 25°C

    (B) C (graphite) is more thermodynamically stable than C(diamond) at 25°C

    (C) graphite will provide more heat on complete combustion at 25°C

    (D) $\Delta G_{transition}$ of $C(diamond) \rightarrow C(graphite)$ is –ve

SPACE FOR ROUGH WORK

8. Consider the following statements regarding Maxwell's distribution of velocities. The correct statement(s) is/are:

(A) As temperature increases, the peak (maxima) of a curve is shifted towards right side

(B) As temperature increases, the most probable velocity of molecules increases but fraction of molecules of maximum velocity decreases.

(C) The area under the curve at all the temperature is the same because it represents the number of gaseous molecules

(D) The fractions of molecules having different velocities are same at a given temperature.

9. Consider the following two molecules and according to the given information select correct statement(s) about AX_2 and AY_2.

Where A: 16th group of 3rd period element

X: more electronegative than (A) and same group number of (A)

Y: Less atomic size than (A) and same period number of (A)

(A) The hybridization of central atoms are different in both compounds

(B) The shape of both molecules are same

(C) Both compounds are planar

(D) The X – A – X bond angle is less than Y – A – Y bond angle

10. There are large number of different receptors in the body that interact selective with different chemical messengers because:

(A) Their binding sites have different shape

(B) Their binding sites have different structure

(C) Their binding sites have different amino acid composition

(D) Their binding sites produce different chemicals

11. Which of the following statement(s) is (are) true?

(A) Maltose on hydrolysis gives two molecules of α – D-Glucose

(B) Starch has two components- Amylose and Amylopectin, Amylose is soluble in water

(C) On oxidation with nitric acid, glucose yield a monocarboxylic acid

(D) Mutarotation is a general property of cyclic (chiral) acetals

12. Which of the following is/are correct?

(A) Aspirin stimulates the synthesis of prostaglandin

(B) Chloramphenicol is used as drug because of its low toxicity for humans

(C) Antibiotic in low concentration inhibits the growth of microorganisms by intervening in their reproductive processes

(D) Phenol can be used as both, an antiseptic as well as a disinfectant depending upon its concentration

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SPACE FOR ROUGH WORK

Section-3 | Numerical Value Type

This **Section** contains **6 Numerical Value Type Questions**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

13. If following molecules undergo dimerization then find the value of $\dfrac{YZ}{X}$.

    (i) ClO_3 **(ii)**   OF   **(iii)** GaH_3 **(iv)** $AlCl_3$

    (v) ICl_3 **(vi)** BeH_2 **(vii)** NO_2

Where : $X =$ Number of molecules which are hypervalent in dimeric form.

$Y =$ Number of molecules which complete octet in dimeric form

$Z =$ Number of molecules which form 3c-2e bonds in dimeric form

14. Choose total number of correct reactions:

    (i)   When $CuSO_4$ solution reacts with NH_3, complex is formed.

    (ii)   When $CuSO_4$ solution reacts with PH_3, complex is formed.

    (iii) $C_{12}H_{22}O_{11} \xrightarrow{\text{conc. } H_2SO_4} 12C + 11H_2O$

    (iv) $NH_3 + Cl_2 \text{(excess)} \xrightarrow{\Delta} NH_4Cl + N_2$

    (v) $NH_3 + Cl_2 \text{(excess)} \xrightarrow{\Delta} NCl_3 + HCl$

    (vi) $HNO_3 + P_4O_{10} \xrightarrow{\Delta} HPO_3 + N_2O_5$

    (vii) $S + H_2SO_4 \xrightarrow{\Delta} SO_2 + H_2O$

    (viii) $SbF_5 + XeF_4 \longrightarrow [SbF_4][XeF_5]$

    (ix) $XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2$

**15.** A solution is saturated in $SrCO_3$ and SrF_2. The CO_3^{2-} was found to be $10^{-3}\,mol/L$. If the concentration of F^- in solution is represented as $y \times 10^{-2}\,M$ then what is the value of y?

[Given : $K_{sp}(SrCO_3) = 2.5 \times 10^{-10}$; $K_{sp}(SrF_2) = 10^{-10}$]

16. If 15.4 g of a solute (molecular weight = 154 g/mole) is dissolved in 260 g of benzene. What will be the boiling point (in °C) of the resulting solution.

[Given $K_b(C_6H_6) = 2.6\,K\ Kg\ mol^{-1}$ and $T_b(C_6H_6) = 80.1°C$]

17. Isomerisation of cyclobutene into 1, 3-butadine follows first order kinetics as:

 (g) ⟶ (g)
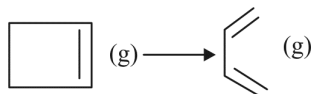
The kinetic study was performed by taking same amounts of cyclobutene in three sealed flasks. First flask was broken after 20 minute and the reaction mixture was absorbed completely in bromine solution. 16.0 mL 1.0 M bromine solution was required. The second flask was broken after a very long time and the reaction mixture required 20 mL bromine solution of the same strength. If the third flask was broken after 40 minutes, what volume of bromine solution of same strength would have been required? (in mL)

18.
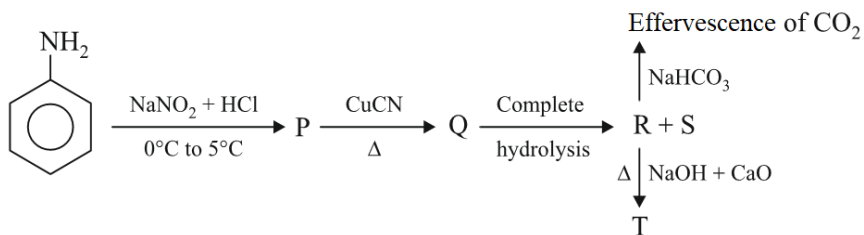
Molecular weight of T will be:

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SUBJECT III : MATHEMATICS   62 MARKS

Section-1 | Single Correct Type

This Section contains **4 Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

1. a and b are two positive real numbers and z_1, z_2 be the two distinct points in argand plane. If

   $a|z_1| = b|z_2|$ then $\left(\dfrac{az_1}{bz_2} + \dfrac{bz_2}{az_1} \right)$ is a point on:

   (A)   a line segment on real axis   (B)   a line segment on imaginary axis

   (C)   a unit circle   (D)   the line $Arg(z) = \tan^{-1} 2$

2. Let A_1, A_2, A_3, ..., A_8 be skew symmetric matrices of same order then the matrix

   $$M = A_1 + 3(A_2)^3 + 5(A_3)^5 + ... + 15(A_8)^{15}$$

   (A)   is symmetric

   (B)   is skew symmetric

   (C)   is neither symmetric nor skew symmetric

   (D)   has $\det(M) = 0$

3. Through a point (p, q) secants are drawn to the circle $x^2 + y^2 = \gamma^2$. If the locus of midpoints of

   chords to the circle is $x^2 + y^2 + 2hxy + 2gx + 2fy + c = 0$ then:

   (A) $g + f = p + q$ (B) $g + f = -p - q$

   (C) $2(g + f) + p + q = 0$ (D) $2(g + f) = p + q$

4. Let An denotes the area formed by the elements of set A, defined below.

   $$A = \left\{ (x, y) \in R \times R; \; [x] + \{x\}^2 \leq y \leq [x] + \sqrt{\{x\}}, \; y \geq 0, \; 0 \leq x \leq n, \; n \in N \right\}.$$

   If $[\cdot]$ denote greatest integer functions and $\{\cdot\}$ denotes the fraction part function, then which of the following is correct?

   (A) $A_1 + A_2 + A_3 + ... + A_{20} = 150$ (B) $A_3 + A_6 + A_9 + ... + A_{30} = 110$

   (C) A_3, A_6, A_9, A_{12} are in H.P.   (D) $A_3 + A_6 + A_9 + ... + A_{30} = 55$

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Section-2 | Multiple Correct Type

This Section contains **8 Multiple Correct Answer Type Questions.** Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

5. Let $Q(x) = x^2 - 2x + 3$ and $P(x) = Q(Q(x)) - x$ then which of the following is/are correct?

(A) $P(x) = 0$ has 2 real roots

(B) $P(x) = 0$ has 4 imaginary roots

(C) sum of real roots $= 1$

(D) sum of reciprocal of imaginary roots $= \dfrac{3}{2}$

6. $A = \begin{bmatrix} 0 & 3 & 2 \\ 0 & -2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ and $4A^{-1} = aA^2 + bA + cI$, where $a, b, c \in$ integers and I is the identity matrix

of order 3, then:

(A) $a = 4$ (B) $a^2 + b^2 + c^2 = 18$

(C) $a + b - c = 6$ (D) $|a| + |b| + |c| = 6$

7. In an experiment, performances of a single throw of unbiased normal dice, let three events E_1, E_2 and E_3 are defined as follows:

E_1 : getting prime numbered face on two dice.

E_2 : getting the same number on two dice.

E_3 : getting 5 as sum on two dice.

then which of the following is true

(A) $p(E_1)$, $p(E_2)$ and $p(E_3)$ are in A.P. (B) $p(E_1)$, $p(E_2)$ and $p(E_3)$ are in G.P.

(C) the events E_3 and E_2 are exclusive (D) $p(E_3 / E_1) = \dfrac{2}{9}$

8. Let $ABCD$ be a rectangle with $AB = a$ and $BC = b$. A circle is drawn passing through points A and B and touching the side CD. Another circle is drawn passing through points B and C and touching side AD. Let r_1 and r_2 be the radii of these two circles respectively then which of the following is true

(A) $r_2 = \dfrac{4b^2 + a^2}{8b}$ (B) $r_1 = \dfrac{4b^2 + a^2}{8b}$ (C) $r_1 = \dfrac{4a^2 + b^2}{8a}$ (D) $r_2 = \dfrac{4a^2 + b^2}{8a}$

9. Consider the following curves.

$$C : y^2 = 4px; \quad D : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1; \quad E : \dfrac{x^2}{5} + \dfrac{y^2}{12} = k^2$$

P is an end point of the latus rectum of C, and the ellipse D passes through the point P, the eccentricity e of the ellipse D is such that the tangents to the parabola and the ellipse at the point P are perpendicular to each other. If $\left(e, \dfrac{b}{a} \right)$ is a point on the ellipse E, then:

(A) $15k^2 = 4$ (B) $a^2 = 3p^2$ (C) $a^2 = 2b^2$ (D) $e = \dfrac{1}{\sqrt{2}}$

10. $g(x) = 3f\left(\dfrac{x^2}{3}\right) + f(12 - x^2),\ \forall x \in R;\ f''(x) > 0\ \forall x \in R,$ then:

(A) $g(x)$ is increasing in $(0, \infty)$

(B) $g(x)$ is increasing in $(-\infty, -3) \cup (3, \infty)$

(C) $g(x)$ is decreasing in $(-\infty, -3)$

(D) $g(x)$ is decreasing in $(0, 3)$

11. Let $f(x)$ and $g(x)$ be two differentiable functions satisfying the following conditions $f(0) = 2,\ g(0) = 1,\ f(x) + g'(x) = f'(x) + g(x) = 1,\ x \geq 0$ then which of the following is/are true?

(A) $f(x) = \dfrac{1}{2}[e^x + e^{-x}]$

(B) $f(x) = \dfrac{1}{2}[e^x + e^{-x}] + 1$

(C) $g(-x) = \dfrac{1}{2}[e^x - e^{-x} + 1]$

(D) $f(x) + g(x) = 2 + e^{-x}$

12. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-coplanar unit vectors and $\vec{b}$ and $\vec{c}$ be non-collinear. If $2\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \vec{c}$ then the angle between:

(A) $\vec{a}$ and $\vec{c}$ is 60°

(B) $\vec{a}$ and $\vec{b}$ is 120°

(C) $\vec{b}$ and $\vec{c}$ is 30°

(D) $\vec{b}$ and $\vec{c}$ is 90°

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Section-3 | Numerical Value Type

This **Section** contains **6 Numerical Value Type Questions**. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

13. Let $\omega \neq 1$ be a cube root of unity and a, b, c are distinct odd integers, then the minimum value of $|a + b\omega + c\omega^2|$ is $2\sqrt{k}$ then the value of k is _____.

14. The first 3 terms of an arithmetic progression are a, b and a^2 where a is negative. If first 3 terms of a GP are a, a^2 and b and the sum of G.P. (upto infinite terms) is $-\dfrac{1}{\sqrt{k}}$ then the value of k is_____.

15. A person either walks to school or takes bus to school everyday. The probability that he takes a bus to school is $\dfrac{1}{4}$. If he takes a bus, the probability that he will be late is $\dfrac{2}{3}$. If he walks to school, the probability that he will be late is $\dfrac{1}{3}$. The probability that the person in on time for at least one out of two consecutive days is $\dfrac{p}{q}$ where p and q are co-prime than the value of $\sqrt{q-p}$ is _____.

16. Let S be a circle with centre $C(-1, 1)$ and PA and PB are pair of tangents drawn from $P(7, 7)$ to touch the circle at A and B. If D is a point equidistant from the vertices of quadrilateral $PACB$ than the distance between point D and origin is _____.

17. If $y = f(x)$ be a function satisfying $f(x) + f\left(1 - \dfrac{1}{x}\right) = x \; \forall x \in R - \{0, 1\}$. Then the value of $a - b$, where $\displaystyle\int_0^1 x(1-x)f(x)dx = \dfrac{a}{3b}$. [$a, b$ are co-prime numbers]

18. If the equation of the plane passing through the point $(2, 3, -4)$ and parallel to the lines $\dfrac{x-2}{2} = \dfrac{y-1}{0} = \dfrac{z+1}{1}$ and $\dfrac{x-1}{1} = \dfrac{y+1}{2} = \dfrac{z-3}{-2}$ is $ax + by + cz + 5 = 0$ then the value of $a + b + c$ is _____.

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